PVP 23

Code: 23BS1101

I B.Tech - I Semester – Supplementary Examinations - JULY 2024

LINEAR ALGEBRA & CALCULUS

(Common for ALL BRANCHES)

Duration: 3 hours Max. Marks: 70

Note: 1. This question paper contains two Parts A and B.
2. Part-A contains 10 short answer questions. Each Question carries 2 Marks.
3. Part-B contains 5 essay questions with an internal choice from each unit. Each Question carries 10 marks.
4. All parts of Question paper must be answered in one place.

PART – A

1.a)	Find the value(s) of k such that the rank of $A=\begin{bmatrix}1 & 2 & 3\\ 2 & k & 7\\ 3 & 6 & 10\end{bmatrix}$ is 2.
1.b)	Find the first approximation to the solution of the following system of equations using Jacobi's method by taking initial approximation as zero $5x+y+2z=19, x+4y-2z=-2,\ 2x+3y+8z=39$
1.c)	If the product of two eigen values of $A=\begin{bmatrix}8 & -2 & 2\\ 0 & 2 & -1\\ 0 & 0 & 4\end{bmatrix}$ is 16 then find the third eigen value.
1.d)	Write down the symmetric matrix of the quadratic form $x_1^2+3x_2^2-2x_3^2+2x_1x_2-6x_1x_3-4x_2x_3$
1.e)	Determine c value where $c\in(1,2)$, for the functions x and x^2 defined in [1, 2] by using Cauchy's mean value theorem.
1.f)	Write the Taylor's series expansion of $f(x)$ about $x=x_0$

1.g)	Discuss the continuity of the function $f(x,y)$ at origin where $f(x,y)=\begin{cases}\frac{5xy}{x^2-y^2}, (x,y)\neq(0,0)\\ 0 \quad\quad , (x,y)=(0,0)\end{cases}$
1.h)	Find the first and second order partial derivatives of $f(x,y)=x^3+y^3-3axy$
1.i)	Write the limits by changing the variables of the double integral $\int_0^{\infty}\int_0^{\infty} 1\ dxdy$ to polar coordinates with the help of region of integration.
1.j)	Evaluate the double integral $\int_0^1\int_0^2 x^2y\ dydx$

PART – B

					Max. Marks
UNIT-I					
2	a)	Reduce the matrix $A=\begin{bmatrix}1&2&3&0\\2&4&3&2\\3&2&1&3\\6&8&7&5\end{bmatrix}$ into echelon form and hence find its rank.			5 M
	b)	Solve the system of non-homogeneous linear equations $x+y+z=6,\ x+2y+3z=14,\ x+4y+7z=30$.			5 M
OR					
3	a)	Apply Gauss Jordan method to find the inverse of the matrix $\begin{bmatrix}1&1&3\\1&3&-3\\-2&-4&-4\end{bmatrix}$			5 M
	b)	Use Gauss Seidel iteration method to solve the system of equations $10x+y+z=12,\ 2x+10y+z=13,\ 2x+2y+10z=14.$			5 M

		UNIT-II	
4	a)	Find Eigen values and corresponding Eigen vectors of the matrix $A=\begin{bmatrix}1 & 1 & 3\\ 1 & 5 & 1\\ 3 & 1 & 1\end{bmatrix}$	5 M
	b)	Make use of the eigen values of matrix of the quadratic form to discuss the rank and nature of the quadratic form $2x_1^2+2x_2^2+3x_3^2+2x_1x_2-4x_1x_3-4x_2x_3$	5 M
		OR	
5	a)	Verify Cayley-Hamilton theorem for $A=\begin{bmatrix}2 & 1 & 1\\ 0 & 1 & 0\\ 1 & 1 & 2\end{bmatrix}$ and then find A^{-1}	5 M
	b)	If the eigen values of a matrix A of order 3 and the corresponding eigen vectors are $1, 1, 3$ & $[1,0,-1]^T$, $[0,1,-1]^T$, $[1,1,0]^T$ respectively then find the matrix A.	5 M
		UNIT-III	
6	a)	Check the applicability of Rolle's theorem for the function $f(x)=\frac{\sin x}{e^x}$ in $\left[0,\frac{\pi}{2}\right]$, if applicable find C value.	5 M
	b)	Write the series expansion of $f(x)=\log(1+x)$ in powers of x.	5 M
		OR	
7	a)	Apply mean value theorem to prove $\frac{b-a}{\sqrt{1-a^2}} < \sin^{-1}b-\sin^{-1}a < \frac{b-a}{\sqrt{1-b^2}}$, $(0<a<b)$. Hence deduce that $\frac{\pi}{6}+\frac{1}{5\sqrt{3}} < \sin^{-1}\left(\frac{3}{5}\right) < \frac{\pi}{6}+\frac{1}{8}$	5 M
	b)	Write series expansion of $f(x)=\tan^{-1}x$ in powers of x by applying suitable series expansion.	5 M

		UNIT-IV	
8	a)	Make use of functional determinant to show $\frac{\partial(x,y)}{\partial(r,\theta)} \times \frac{\partial(r,\theta)}{\partial(x,y)} = 1$ where $x = r\cos\theta, y = r\sin\theta$	5 M
	b)	Discuss the nature of stationary points and then find extreme values of $f(x,y) = x^3 + y^3 - 63(x+y) + 12xy$	5 M
		OR	
9	a)	If $U = \frac{1}{\sqrt{x^2+y^2+z^2}}$, $x^2+y^2+z^2 \neq 0$, then prove that $\frac{\partial^2 U}{\partial x^2} + \frac{\partial^2 U}{\partial y^2} + \frac{\partial^2 U}{\partial z^2} = 0$	5 M
	b)	A rectangular box which is open at the top, is to have volume 32 cubic ft. Find the dimensions of the rectangular box requiring least material for its construction.	5 M
		UNIT-V	
10	a)	By changing the order of integration, evaluate the double integral $\int_0^1 \int_{x^2}^{2-x} xy \; dydx$	5 M
	b)	Determine the volume of the solid bounded by the planes $x = 0, y = 0, x + y + z = 2 \; and \; z = 0.$	5 M
		OR	
11	a)	Calculate the value of triple integral in cartesian coordinates $\int_0^1 \int_1^2 \int_2^3 (x^2+y^2+z^2)\, dz\, dy\, dx$	5 M
	b)	Find the area enclosed by the pair of curves $y = 2 - x$ and $y^2 = 2(2-x)$ using double integration.	5 M